Assignment

Paper – C201 :: Sem – 2nd :: Course – Mathematics (H) UG

Subject: Groups Theory and Vector Analysis – I

Vector Analysis – I

1. State and prove Lami's theorem using vector method.
2. If the four forces acting at a point are in equilibrium then each force is proportional to the volume of the parallelopiped determined by unit vectors in the directions of other three.
3. The angular velocity of a rotating rigid body about an axis of rotation is given by $(4\hat{i}+\hat{j}-2\hat{k})$. Find the linear velocity of a point on the rigid body whose position vector relative to a point on the axis of rotation is $(2\hat{i}-3\hat{j}+\hat{k})$.
4. Show that the torque about the point $(2\hat{i}+\hat{j}-3\hat{k})$ of a force represented by $(\hat{i}+2\hat{j}+\hat{k})$ passing through the point $(3\hat{i}+4\hat{j}-\hat{k})$ is $(\hat{j}-\hat{i}-\hat{k})$.
5. The necessary and sufficient condition that the vector equation $\vec{a}\times\vec{x}=\vec{b}$ where $\vec{a}$ & $\vec{b}$ are given vectors and $\vec{a}\neq 0$ possesses a solution is that $\vec{a}\cdot\vec{b}=0$
6. Show that the solution of the equation $k\vec{r}+\vec{r}\times\vec{a}=\vec{b}$ where k is non zero scalar and $\vec{a}$ & $\vec{b}$ are two vectors can be put as $\vec{r}=\frac{1}{k^2+|\vec{a}|^2}\left[\frac{\vec{a}\cdot\vec{b}}{k}\vec{a}+k\vec{b}+\vec{a}\times\vec{b}\right]$
7. The necessary and sufficient condition for a vector $\vec{r}=\vec{f}(t)$ to have constant direction is $\vec{f}\times\frac{d\vec{f}}{dt}=0$
8. State and prove Frenet Serret formula.
9. Show that for plane curve $\tau=0$.
10. Show that the acceleration $\vec{a}$ of a particle which travells along a space curve with velocity $\vec{v}$ is given by $\vec{a}=\frac{dv}{dt}\vec{t}+\frac{v^2}{\rho}\vec{n}$ where $\vec{t}$ is the unit tangent vector to the space curve, $\vec{n}$ is the unit principal normal and ρ is the radius of curvature.
11. Find the equations of the oscillating plane, normal plane and rectifying plane to the cubic $x=2t$, $y=t^2$, $z=t^3$ at $t=1$
12. Show that the curve $\vec{r}=\vec{r}(s)$ is a helix ($\frac{\kappa}{\tau}=$ constant) iff $\frac{d^2\vec{r}}{ds^2}\cdot\frac{d^3\vec{r}}{ds^3}\times\frac{d^4\vec{r}}{ds^4}=0$, where s is the length of the curve.
13. Show that the equation of the tangent line to the curve $x=t$, $y=t^2$, $z=\frac{2}{3}t^3$ at $t=1$ are $2(x-1)=y-1=z-\frac{2}{3}$
15. If $\vec{r}(t)=2\hat{i}-\hat{j}+2\hat{k}$ when $t=2$ and $\vec{r}(t)=4\hat{i}-2\hat{j}+3\hat{k}$ when $t=3$ then show that $\int_2^3\left(\vec{r}\cdot\frac{d\vec{r}}{dt}\right)dt=10$.
16. Show that the volume of a tetrahedron bounded by four planes $\vec{r}\cdot(m\hat{j}+n\hat{k})=0$, $\vec{r}\cdot(n\hat{k}+l\hat{i})=0$, $\vec{r}\cdot(l\hat{i}+m\hat{j})=0$ and

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$\vec{r}\cdot(l\hat{i}+m\hat{j}+n\hat{k}) = p$ is $\dfrac{2p^3}{3lmn}$,

17. Show that the distance of a corner of a unit cube from the diagonal not passing through it is $\frac{1}{3}\sqrt{6}$ unit.

18. Show that the condition that the three planes $\vec{r}\cdot\vec{n_1} = p_1$, $\vec{r}\cdot\vec{n_2} = p_2$ and $\vec{r}\cdot\vec{n_3} = p_3$ should have common line of intersection is $p_1(\vec{n_2}\times\vec{n_3}) + p_2(\vec{n_3}\times\vec{n_1}) + p_3(\vec{n_1}\times\vec{n_2}) = \vec{0}$

19. Find the vector equation of the plane through the point $(\hat{i}+2\hat{j}+\hat{k})$ and perpendicular to the line of intersection of the planes $\vec{r}\cdot(3\hat{i}-\hat{j}+\hat{k}) = 1$ and $\vec{r}\cdot(\hat{i}+4\hat{j}-2\hat{k}) = 2$

20. Find the equation of straight line through the point $\vec{c}$ which parallel to the plane $\vec{r}\cdot\vec{a} = 0$ and intersects the line $\vec{r} = \vec{a} + t\vec{b}$.

21. Prove that the line $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$ intersect and find their point of intersection.

22. Find a unit vector in the plane of vectors $\vec{\alpha}$ and $\vec{\beta}$ and is $\perp^r$ to $\vec{\gamma}$ where $\vec{\alpha} = \hat{i}+2\hat{j}-\hat{k}$, $\vec{\beta} = \hat{i}+\hat{j}-2\hat{k}$ and $\vec{\gamma} = 2\hat{i}-\hat{j}+\hat{k}$.

Group theory–1

1. Let a be an element of a group $(G,\circ)$ then prove that
   (i) if $o(a) = n$ and $a^m = e$ then n is a divisor of m.
   (ii) if $o(a) = n$ then $a, a^2, \ldots, a^n (= e)$ are distinct elements of G.
   (iii) if $o(a) = n$ then for a positive integer m, $o(a^m) = \dfrac{n}{\gcd(m,n)}$

2. Prove that order of an element in a finite group can not exceed the order of a group.

3. In a group $(G,\circ)$, the elements a and b are commute and $o(a)$ and $o(b)$ are prime to each other. Then show that $o(a\circ b) = o(a)\cdot o(b)$.

4. Prove that any conjugate of a has the same order as that of a. Hence deduce that $o(a\circ b) = o(b\circ a)$ $\forall a,b\in G$ (group).

5. In $(G,\circ)$ be a group in which $(a\circ b)^3 = a^3\circ b^3$ and $(a\circ b)^5 = a^5\circ b^5$ $\forall a,b\in G$. Prove that G is an abelian group.

6. In a group G, $a^{n+1}b^{n+1} = b^{n+1}a^{n+1}$ and $a^n b^n = b^n a^n$ hold for all a, b in G and $\forall n \geq 2$. Prove that G is an abelian group.

7. Let $(G,\circ)$ be a group and $a,b\in G$. If $o(a) = 3$ and $a\circ b\circ a^{-1} = b^2$ find $o(b)$ if $b \neq e$.

8. Prove that union of two subgroups of a group is subgroup under what condition. Justify.

9. Let $(G,\circ)$ be a group and H be a nonempty finite subset of G. Then $(H,\circ)$ is a subgroup of $(G,\circ)$ iff $a\in H, b\in H \Rightarrow a\circ b\in H$.

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10. Let H and K be finite subgroups of a group G such that HK is a subgroup of G. Then prove that $O(HK) = \dfrac{O(H)\,O(K)}{O(H \cap K)}$.

11. Prove that every subgroup of a cyclic group is cyclic.

12. Find all cyclic subgroups of the symmetric group S_3

13. Prove that $(\mathbb{Q},+)$ is a non-cyclic group. Hence deduce that $(\mathbb{R},+)$ is non-cyclic.

14. Let n be a positive integer and let S be the set of nth roots of unity. Show that $(S,\cdot)$ is a cyclic group. Find all generators.

15. Prove that any two right cosets of a subgroup are either disjoint or identical.

16. State and prove Lagrange's Theorem. Is the converse of the theorem true? Justify.

17. Let G be group and H be a subgroup of G. Let $a \in G-H$. Prove that $aH \cap H = \phi$

18. Prove that a subgroup H of a group G is normal iff $xHx^{-1} = H \; \forall x \in G$.

19. Prove that Centre $Z(G)$ of a group G is a normal subgroup of G.

20. If A and B are normal subgroup of G, Prove that AB is also normal subgroup of G.

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