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B.Sc. (Mathematics)

(Hons)

4th Sem.

Paper - CC10

~~Assignment~~

Answer all questions:

1. Show that the vectors $(1, 2, 1)$, $(2, 1, 5)$ and $(3, 4, 7)$ are L.I in $\mathbb{R}^3$
2. Define "norm" of a vector in a real inner product space. Prove that any orthogonal set of non-null vectors in an inner product space is L.I.
3. Show that $\alpha = (1, 2, -1, -2)$, $\beta = (2, 3, 0, -1)$, $\gamma = (1, 2, 1, 4)$ and $\delta = (1, 3, -1, 0)$ form a basis of the vector space V_4 over the real numbers.
4. Show that the set $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ is a basis of $V_3(\mathbb{R})$. Find the coordinates of vectors (a, b, c) with respect to the above basis.
5. Use Gram-Schmidt process to obtain an orthogonal basis from the basis set $\{(1, 2, -2), (2, 1, 1), (1, 1, 0)\}$ of the Euclidean space $\mathbb{R}^3$ with standard inner product.
6. Prove that any two bases of a finite-dimensional vector space have the same number of elements.
7. Prove that the rank of the product of two matrices cannot exceed the rank of either factor.
8. Find the basis for the vector space $\mathbb{R}^3$ that contain the vectors $(1, 2, 0)$, $(1, 3, 1)$.

Assignment

9. Show that $W = \{ (x, y, z) \in \mathbb{R}^3, x+y+z=0 \}$ is a Subspace of $\mathbb{R}^3(\mathbb{R})$. Find a basis of W .
10. If $\alpha, \beta \in \mathbb{R}^2(\mathbb{R})$, then show that the set $\{ \alpha, \beta, \alpha + \beta \}$ where $\alpha, \beta \in \mathbb{R}$ is dependent.
11. Show that $W = \{ (x, y, z) \in \mathbb{R}^3 : x+2y=z, 2x+3z=y \}$ is a Subspace of $\mathbb{R}^3(\mathbb{R})$. Find a basis of W .
12. Show that the set $S = \{ (1,0,0), (0,1,0), (0,0,1), (1,1,1) \}$ spans $\mathbb{R}^3$ but not a basis of $\mathbb{R}^3$.
13. Prove that every vector space has a basis.
14. Use Gram-Schmidt method to obtain an orthogonal basis of $\mathbb{R}^3$ from $\{ (1,1,1), (0,1,2), (2,1,1) \}$.
15. In a Euclidean space V , if α, β be two L.I vectors, then $|\langle \alpha, \beta \rangle| < \|\alpha\| \|\beta\|$.
 (b) Any orthogonal finite set of non-null vectors in V is L.I.
16. Extend $(2,3,-1), (1,-2,-4)$ to an orthogonal basis of $\mathbb{R}^3$ and then find the associated orthonormal basis.