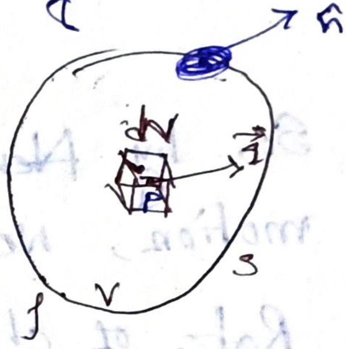
Non Newtonian :

A fluid is said to be Non newtonian if it's viscosity μ varies with the rate of deformation i.e, τ, μ, $\frac{du}{dy}$ all are variables in the eqn $\tau = \mu \frac{du}{dy}$, and this type of fluid is represented by curve line.

(Navier - Stoke's ~~theorem~~ eqn) (N-S) for viscouss fluid :

Let us consider an arbitrary closed surface S drawn in a domain occupied by a viscous fluid and moving with it.



Let ρ & $\vec{q}$ denotes the density, velocity of fluid at P within a surface S and dv be the elementary volume enclosing P.

Let $\hat{n}$ be unit outword normal vector on dS. Then the linear momentum of the element is $\rho \vec{q}\, dv$.

$\therefore$ $\vec{M}$ = total linear momentum

$$= \sum \rho \vec{q}\, dv$$

$$= \int_V \rho \vec{q}\, dv$$

$$\frac{d\vec{M}}{dt} = \frac{d}{dt}\int_V \rho\vec{q}\,dv$$

$$= \int_V \vec{q}\,\frac{d}{dt}(\rho\,dv) + \int_V \frac{d\vec{q}}{dt}(\rho\,dv)$$

$$= \int_V \frac{d\vec{q}}{dt}\,\rho\,dv \quad \left(\because \frac{d}{dt}(\rho\,dv) = 0\right)$$

Let F be the external force per unit mass acting on the element and $P(\hat{n})$, the shearing stress on the element ds

Where $P(\hat{n}) = P_x n_x + P_y n_y + P_z n_z$

$\hat{n} = \hat{n}(n_x, n_y, n_z)$

So, by ~~Newot's~~ Newton's 2nd law of motion, we have

Rate of change of linear momentum = Total acting forces.

$$\therefore \int_V \frac{d\vec{q}}{dt}\,\rho\,dv = \int_V \vec{F}\rho\,dv + \int_S P(\hat{n})\,ds$$

or, $$\int_V \left(\frac{d\vec{q}}{dt} - \vec{F}\right)\rho\,dv = \int_S (n_x P_x + \ldots + n_z \ldots$$

$$= \int_V \left(\frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z} \ldots\right.$$

(, by Gauss div. t...

$$\int_V \vec{\nabla} P(n) \, dv = \int_V \vec{\nabla}\cdot\vec{P} \, dv,$$

, where $\vec{P}$ is the stress tensor

$$\Rightarrow \int_V \left[\rho\left(\frac{d\vec{q}}{dt} - F\right) - \vec{\nabla}\cdot\vec{P}\right] dv = 0$$

Since dv is an arbitrary,

$$\therefore \boxed{\frac{d\vec{q}}{dt} = F + \frac{1}{\rho}\vec{\nabla}\cdot\vec{P}} \quad \text{(1)}$$

Since $\vec{P}$ is the stress tensor, so, we have

$$\vec{\nabla}\cdot\vec{P} = \sum \frac{\partial}{\partial x_j}(P_{ij})$$

$$= \sum_i \frac{\partial}{\partial x_j}\left[2\mu T_{ij} - \left(p + \frac{2}{3}\mu\vec{\nabla}\cdot\vec{q}\right)\delta_{ij}\right]$$

$$= \sum_j \left[2\mu \frac{\partial T_{ij}}{\partial x_j} - \frac{\partial}{\partial x_i}\left(p + \frac{2\mu}{3}\vec{\nabla}\cdot\vec{q}\right)\right] \quad \text{(2)}$$

$$= 2\mu \sum_i \frac{\partial T_{ij}}{\partial x_j} - \left[\vec{\nabla} p + \frac{2\mu}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right]$$

$$2\sum \frac{\partial T_{ij}}{\partial x_j} = \sum_j \frac{\partial}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$= \sum_i \frac{\partial^2 u_i}{\partial x_j \partial x_i} + \sum_i \frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_i}\right)$$

$$= \nabla^2 u_i + \frac{\partial}{\partial x_i}(\vec{\nabla}\cdot\vec{q})$$

$$= \nabla^2\vec{q} + \vec{\nabla}(\vec{\nabla}\cdot\vec{q}).$$

From (2) we have,

$$\vec{\nabla}\cdot\vec{P} = \mu\left[\nabla^2\vec{q} + \vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right] - \left[\vec{\nabla}p + \frac{2}{3}\mu\vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right]$$

$$= \mu\left[\nabla^2\vec{q} + \frac{1}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right] - \vec{\nabla}p.$$

Then eqn (1) becomes,

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{\rho}\left[\nabla^2\vec{q} + \frac{1}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right]$$

$$= \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu\left[\nabla^2\vec{q} + \frac{1}{3}\vec{\nabla}(\vec{\nabla}\cdot\vec{q})\right]$$

on -- viscous fluid, we have

$\vec{\nabla}\cdot\vec{q} = 0$

$\therefore$ then eqn of motion becomes,

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{q}.$$

se 1: The components ~~from~~ form of eqn (3) in Cartesian co-ordinate are $\vec{q} = (u, v, w)$, $\vec{F} = (X, Y, Z)$.

$$-- = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u + \frac{1}{3}\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y} + \frac{\partial^2 w}{\partial x\partial z}\right\}\right]$$

$$= Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)v + \frac{1}{3}\left(\frac{\partial^2 u}{\partial y\partial x} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y\partial z}\right)\right]$$

$$= Z - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)w + \frac{1}{3}\left(\frac{\partial^2 u}{\partial z\partial x} + \frac{\partial^2 v}{\partial y\partial z} + \frac{\partial^2 w}{\partial z^2}\right)\right]$$

2: Eq^n (3) in cylindrical co-ordinate (r, θ, z).

$$\frac{du}{dt} = -\frac{\partial}{\partial r}\left(\Omega + \frac{p}{\rho}\right) + \nu\left[\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right]$$

$$\frac{dv}{dt} + \frac{uv}{r} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\Omega + \frac{p}{\rho}\right) + \nu\left[\nabla^2 v + \frac{2}{r^2}\frac{\partial u}{\partial \theta} - \frac{v}{r^2}\right]$$

$$\frac{d}{dt} = -\frac{\partial}{\partial z}\left(\Omega + \frac{p}{\rho}\right) + \nu\nabla^2 w.$$

3: Eq^n (3) in spherical co-odi (r, θ, φ) in ...vative field force and incompressible fluid.

$$-\frac{(v^2 + w^2)}{r} = -\frac{\partial}{\partial r}\left(\Omega + \frac{p}{\rho}\right) + \nu\left[\nabla^2 u - \frac{2u}{r^2} - \frac{2u\,d\theta}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta} - \frac{2}{r^2\sin\theta}\frac{\partial w}{\partial \varphi}\right]$$

$$-\frac{w^2}{r}\cot\theta + \frac{uv}{r} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\Omega + \frac{p}{\rho}\right) + \nu\left[\nabla^2 v - \frac{u}{r^2\sin\theta} + \frac{2}{r^2}\frac{\partial u}{\partial \theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial w}{\partial \varphi}\right]$$

$$+\frac{uw}{r^2} + \frac{uw}{r}\cot\theta = -\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \varphi}\left(\Omega + \frac{p}{\rho}\right) + \nu\left[\nabla^2 w - \frac{w}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial u}{\partial \varphi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u}{\partial \varphi}\right]$$

...re $\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{q}\cdot\vec{\nabla})$.

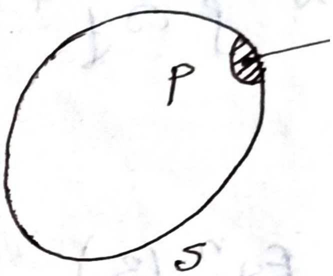
$$= \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta} + \frac{w}{r\sin\theta}\frac{\partial}{\partial \varphi}.$$

And $\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta}$

$$+ \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2}$$

Dissipation of Energy due to viscosity:

Let $\hat{n}$ be the outward normal at any point P on S, and T be the total energy (K.E.).



$\therefore \cancel{T = \frac{1}{2}m}$

$$T = \sum \frac{1}{2}(\rho\, dv)\, |\vec{q}|^2$$

$$= \frac{1}{2}\rho \int_V |\vec{q}|^2\, dv$$

$$\therefore \frac{dT}{dt} = \rho \int_V \left(\vec{q}\cdot\frac{d\vec{q}}{dt}\right) dv$$

$$= \rho \int_V \vec{q}\cdot\left[\vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{q}\right]$$

, where eqn of motion of incompressible viscous fluid,

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu\cdot\nabla^2\vec{q}$$

$$\frac{dT}{dt} = \rho \int_V \vec{q}\cdot\vec{F}\,dv - \int \vec{q}\cdot\vec{\nabla}p\,dv + \mu \int_V \vec{q}\cdot\nabla^2\vec{q}\,d$$

$$\left[\because \nu = \frac{\mu}{\rho}\right.$$

But from the vector identity,

$$\vec{\nabla}\cdot(p\vec{q}) = p\,\vec{\nabla}\cdot\vec{q} + (\vec{q}\cdot\vec{\nabla})p$$

$$= 0 + (\vec{q}\cdot\vec{\nabla})\,p.$$

and $\vec{\nabla}\times(\vec{\nabla}\times\vec{q}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{q}) - \nabla^2\vec{q}$

$$= -\nabla^2\vec{q}$$

Hence above eqn reduces to,

$$\frac{dT}{dt} = \int_V \vec{q}\cdot\vec{F}\,\rho\,dv - \int_V \vec{\nabla}\cdot(p\vec{q})\,dv$$

$$-2\mu\int \vec{q}\cdot(\vec{\nabla}\times\vec{w})\,dv.$$

$$\left(\because \vec{w} = \frac{1}{2}\vec{\nabla}\times\vec{q}\right.$$

~~But $\vec{\nabla}\cdot(\vec{q}\times\vec{w}) = \vec{w}\cdot(\vec{\nabla}\times\vec{q}) - \vec{q}\cdot($~~

But $\vec{\nabla}\cdot(\vec{q}\times\vec{w}) = \vec{w}\cdot(\vec{\nabla}\times\vec{q}) - \vec{q}\cdot(\vec{\nabla}$

$$= w\cdot 2\vec{w} - \vec{q}\cdot(\vec{\nabla}\times\vec{w}$$

Hence $\frac{dT}{dt} = \int_V \vec{q}\cdot\vec{F}\,dv - \int_S \hat{n}\cdot(p\vec{q})\,ds$

$$-\left[4\mu\int_V w^2\,dv - 2\mu\int \hat{n}(\right.$$

Now, the rate of work done per unit mass of the fluid is $= \frac{\text{Work}}{\text{time}}$

$= \frac{\text{Force} \times \text{displacement}}{\text{time}}$

$= \text{Force} \times \text{velocity}$

So, $\int \vec{q} \cdot \vec{F} \, dv$ represents the rate at which ~~press~~ the external force $\vec{F}$ is doing work through out the mass of the liquid;

The term $- \int_S \hat{n} \cdot (p \vec{q}) \, ds$ represents is the rate at which pressure p is doing work on the boundary.

So, from (2), it is clear that the rate at which kinetic energy is being increased by the action of stresses on the surface of the liquid.

Thus, if D is the rate of dessipation of energy due to viscos then from (2) we have

$$D = 4\mu \int_V w^2 dv - 2\mu \int_S \hat{n}\cdot(\vec{q}\times\vec{w})\, ds \quad (3)$$

, where S is the total surface enclosing volume V.

If the boundary is at rest and there is no slip between the fluid and boundary.

Then $\vec{q} = 0$, on S.

Hence $\boxed{D = 4\mu \int_V \vec{w}^2 dv.}$

$$\boxed{\text{i.e., } D = 4\mu \iiint (w_x^2 + w_y^2 + w_z^2)\, dx\, dy\, dz}$$

Circulation of the viscous fluid:

Let Γ be the circulation around the closed path C, then we have

$$\Gamma = \int_C \vec{q}\cdot d\vec{r}.$$

$$\therefore \frac{d\Gamma}{dt} = \int_C \frac{d\vec{q}}{dt}\cdot d\vec{r} + \int_C \vec{q}\cdot\frac{d}{dt}(d$$

$$= \int_C \frac{d\vec{q}}{dt}\cdot d\vec{r} + \int \vec{q}\cdot d\vec{q}$$

$$= \int_C \frac{d\vec{q}}{dt}\cdot d\vec{r} + \frac{1}{2}\int_C d|\vec{q}$$

Now, $\int_C d|\vec{q}|^2 = \left[|\vec{q}|^2\right]_C = 0$

Since C is a closed path and $|\vec{q}|^2$ is a single valued funn.

So we have

$$\frac{d\Gamma}{dt} = \int_C \frac{d\vec{q}}{dt} \cdot d\vec{r}.$$

But the Neivers - Stoke eqn for incompressible fluid under conservative field of force, is

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{q}$$

$$= -\vec{\nabla}\Omega - \frac{1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{q}$$

i.e, $$= -\vec{\nabla}\left(\Omega + \frac{p}{\rho}\right) + \nu\vec{\nabla}^2\vec{q}$$

So, $$\frac{d\vec{q}}{dt} \cdot d\vec{r} = -\vec{\nabla}\left(\Omega + \frac{p}{\rho}\right) \cdot d\vec{r} + \nu\nabla^2\vec{q} \cdot d\vec{r}$$

$$= -d\left(\Omega + \frac{p}{\rho}\right) + \nu\nabla^2\vec{q}. \quad (1)$$

$\vec{\nabla}f \cdot d\vec{r}$

$= \sum \frac{\partial f}{\partial x} dx$

$= df$

From ①, we have

$$\frac{d\Gamma}{dt} = -\left[\Omega + \frac{p}{\rho}\right]_c + \nu \oint_c \nabla^2 \vec{q} \cdot d\vec{r}$$

$$= \nu \int_c \nabla^2 (\vec{q} \cdot d\vec{r})$$

$$= \nu \nabla^2 \int_c \vec{q} \cdot d\vec{r}$$

$$= \nu \nabla^2 \Gamma.$$

Reynold's Number :

The important forces in ~~the~~ a system of flow of a solid in fluid are —

(i) Internal forces of the type $\rho \frac{\partial u}{\partial t}$ or, $\rho u \frac{\partial u}{\partial x}$,

(ii) The frictional forces of the type $\mu \frac{\partial^2 u}{\partial x^2}$.

In the flow of solid through a fluid, the velocities are all proportional to the velocity of the body, say u.

Let us take a length l associated with the body representing the linear scale of measurement keeping the shape of the body fixed.

we can vary l to signify changes in its size. Then the ratio of the internal forces to the viscouss forces

or
$$\frac{\text{internal forces}}{\text{viscouss forces}} = \frac{\rho u^2/l}{\mu u/l^2}$$

$$= \frac{\rho u l}{\mu}$$

$$= \frac{l u}{\nu}$$

$$\boxed{\begin{aligned} &\rho \frac{\partial u}{\partial t} \text{ or } \rho u \frac{\partial u}{\partial x}. \\ &\rho \frac{u}{t} = \rho \frac{u^2}{l}. \\ &\qquad \therefore u = l/t. \\ &\mu \frac{\partial^2 u}{\partial x^2} = \mu \frac{u}{l^2} \end{aligned}}$$

, which is a non dimensional quantity and known as Reynold's Number and denoted by

$$\cancel{Re = \ldots} \quad Re = \frac{ul}{\nu}, \ \nu = \frac{\mu}{\rho}.$$

Hence for geometrical similarities of two flows, it is necessary that their ~~...~~ Reynold's Number should be the same and the boundary conditions are satisfied.

When Reynold's Number is small, the viscouss ~~fo~~ force is predominant and the effect of viscosity is important in the whole velocity field.

When Reynold's Number is large, the inartial force is predominand and the effect of viscosity is effected only in the narrow region near the solid lain which give rise to ~~pr~~ Prandtl ~~B~~ Boundary Lain.

When Reynold's Number is enormously large the flow becomes ~~tur~~ turbulent.

Significance of Reynold's Number :

1. Two flows of incompressible viscous fluid about similar geometrical bodies are dianar similar when Reynold's Number for the flows are equal.

2. Reynold's Number throws light on important ~~fitwre of~~ feature of a given flow. Thus a small Reynold's Number implies that viscosity is predominant where as a large Reynold's Number implies that viscosity is small.

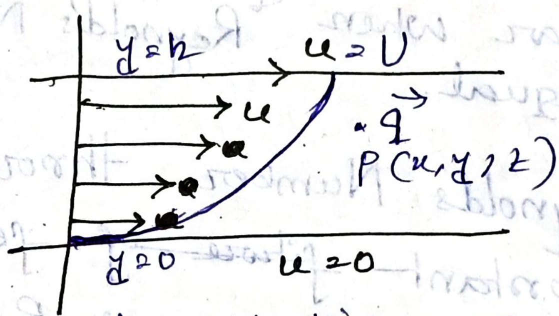
3. It is experimentally shown that if the value of Reynold's Number exceeds a certain critical value (= 2800). the flows ceases to be laminar an it becomes turbulant. When Re < 2 the flow is laminar.

4. The concept of laminar boundary lair was developed by examining the flow for which R_e is very large.

5. Concept of very slow motion or creeping motion was developed by examining the flow for which Re is very small.

Steady flow between $\parallel^{l}$ planes

The motion of viscous fluid of uniform density between $\parallel^{l}$ planes, the motion being steady where one plan is at rest and the other is in motion



Let us incompressible viscous fluid be in steady motion bounded by the planes $y=0$ and $y=h$.

Let the plane $y=0$ i.e, x axis be at rest, while the plane $y=h$ has velocity along x axis. If $\vec{q}$ be the velocity of the fluid at any point then, we have

$$\vec{q} = \vec{q}\,(u,0,0) \quad \text{——— (}$$

Such type of flow is called plane Couette flow.

The equation of Continuity for this case becomes

$$\vec{\nabla}\cdot\vec{q} = 0$$

or, $$\frac{\partial u}{\partial x} = 0 \quad \text{——— (2)}$$

$\Rightarrow$ u is independent of x

So by symmetry, u is also indepependent of z.

$\therefore$ $u = u(y)$ ——— (3)

Now the Neiviers-Stoke's eqn, in absence of body forces becomes,

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{q}$$

$$\Rightarrow \frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\vec{\nabla})\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{q} \quad \text{——— (4)}$$

Since motion is steady

so, $$\frac{\partial\vec{q}}{\partial t} = 0$$

and $$(\vec{q}\cdot\vec{\nabla})\vec{q} = u\frac{\partial(u\hat{i})}{\partial x}.$$

So we have from (4).

$$u\frac{\partial}{\partial x}(u\hat{i}) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right) + \nu\frac{\partial^2}{\partial y^2}(u\hat{i})$$

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \quad \text{——— (5a)}$$

$$\frac{\partial p}{\partial y} = 0, \quad \text{——— (5b)}$$

$\frac{\partial p}{\partial z} = 0$. ———— (5c)

From (5b) and (5c), p is independent of both y and z.

So, p must be function of x

then (5.a) can be writen as,

$$u \cdot 0 = -\frac{1}{\rho}\frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow \boxed{\frac{dp}{dx} = \mu \frac{d^2 u}{dy^2}} \quad \text{———— (6)}$$

From (6), it is clear that, l.h.s. is function of x only, while r.h.s. is function of y only.

Hence each side is constant as the liquid is moving in the +ve direction of x axis, so the pressure $p = p(x)$ should decrease as y increases

i.e, $\frac{dp}{dx} < 0 \quad \forall x > 0$.

Hence (6) can be writen as,

$$\frac{dp}{dx} = \mu \frac{d^2 u}{dy^2} = -P, \text{ say}$$

where $P > 0$.

$$\Rightarrow \frac{d^2 u}{dy^2} = -\frac{P}{\mu}.$$

Integrating, $\frac{du}{dy} = -\frac{P}{\mu} y + A$.

Again integrating,

$$y = -\frac{P}{2\mu} y^2 + Ay + B \quad \text{——— (7)}$$

Using the boundary conditions,

$y=0$, $u=0$ and $y=h$, $u=U$, say.

Then we, have.

$$B = 0, \quad A = \frac{U}{h} + \frac{hP}{2\mu}. \text{ (Calculation)}$$

$$\text{Hence} \quad u = \frac{-Py^2}{2\mu} + \left(\frac{U}{h} + \frac{hP}{2\mu}\right)y \quad \text{——— (8)}$$

Eq^n (8) shows the velocity profile between two planes. and which represent a parabolic profile.

The flow Q per unit bredth is given by

$$Q = \int_0^h u\,dy = \int_0^h \left[-\frac{Py^2}{2\mu} + \left(\frac{U}{h} + \frac{hP}{2\mu}\right)y\right] dy$$

$$=$$

$$=$$

$$= \frac{h^3}{12\mu}P + \frac{1}{2}hU \quad \text{——— (9)}$$

Case I : If both the planes $y=0$ and $y=h$ are at rest, then by putting $U=0$

we get $u = -\frac{P}{2\mu}y^2 + \frac{h}{2\mu}Py$. and

$$Q = \frac{h^3 P}{12\mu}.$$

Case II :

Mean velocity accross such section is $\frac{Q}{h} = \frac{1}{h}\int_0^h u\,dy$

$$\frac{Q}{h} = \frac{h^2}{12\mu}P + \frac{1}{2}U$$

Also the tangential stress at any point (x, y, z) is

$$\mu\frac{du}{dy} = -Py + \frac{\mu U}{h} + \frac{hP}{2}$$

(Diff. 8).

Case III :

Drag per unit area on the lower plane $= \left[\mu\frac{du}{dy}\right]_{y=0}$.

$$\therefore \text{Drag} = \left[\mu\frac{du}{dy}\right]_{y=0}$$

$$= \frac{\mu U}{h} + \frac{hP}{2}$$

And Drag per unit area on the upper plane $\left[\mu\frac{du}{dy}\right]_{y=h} = -\frac{hP}{2} + \frac{\mu U}{h}$