Beta Decay

Both positive and negetive electrons are emitted spontaneously from radioactive nuclei. This phenomenon is called β-decay. The reverse process is the electron capture where the nucleus absorbs one of its own orbital electrons. Like α-decay, β-decay is a process whereby a nucleus can alter its composition to become more stable. Also like alpha decay, β-decay has some puzzling aspects.

β-ray spectrum :- The β-particles ejected from a radioactive source possess a range of velocities and hence a range of energies. The distribution or spread of these energies of β-particles is called the β-ray spectrum of the given nuclide. This distribution can be studied experimentally by deflecting them in a magnetic field. Essentially the track of β-particle in a magnetic field is circle.

if v be the velocity of the β-particle, B the magnetic flux density, m the relativistic mass of the β-particle are r the radius of the circular arc.

Then,

$$\frac{mv^2}{r} = Bev$$

$$\therefore \; mv = Ber$$

$$\text{or,} \quad r = \frac{mv}{Be} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{v}{Be}$$

where m_0 is the rest mass of the β-particle. Now momentum of the emitted β-particle is given by $p = mv = Ber$. Since, $p \propto r$, the particles with higher velocities have larger radii.

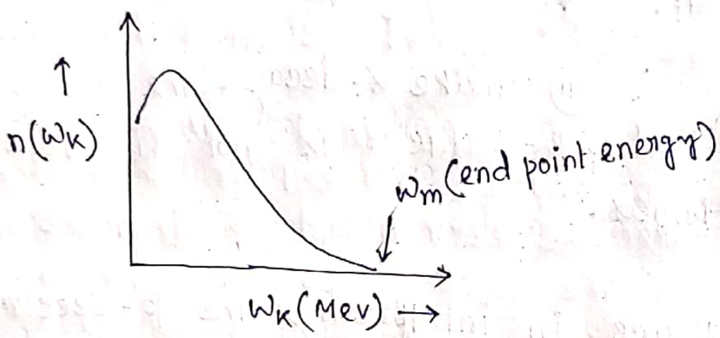
If E be the total energy of the β-particles

$$E^2 = p^2c^2 + m_0^2c^4$$

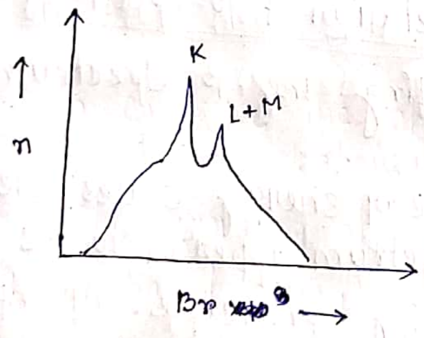
$$\therefore \text{kinetic energy, } \omega_K = E - m_0c^2$$

$$= \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$$

The distribution of the kinetic energy ω_K of the β-particles with the number of particles $n(\omega_K)$ is drawn. This is called the ~~continous~~ continuous β-spectrum.



Instead of W_K, if a graph is plotted between Br ($P = Ber$) and no. of particle (i.e momentum distribution) shows a number of sharp peaks K, L, etc supperimposed on the continuous back ground - the so called line spectrum of the β-rays.



momentum distribution

- **Nature of spectrum** :- The magnetic deflection experiments with various β-emmitters show that a single source produces β-particles with all energies from zero up to a definite maximum W_m, characteristic of the nuclide, the so called end-point energy. This is the continues-β-spectrum, the shape of which is generally the same for all nucleus.

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with this continuous spectrum, there is a number of sharp lines (peaks) which are found to be very prominent. This is the line spectrum of the β-rays and it indicates that there are definite energy levels in the nucleus.

Difference in α-decay and β-decay :- The process of β-decay differs from α-decay in two important respects.

i) α-particle is composed of nucleons already present in the initial nucleus. But electron is not present in the nucleus and must be created in the decay process itself.

ii) unlike α-decay, the energy spectrum of the emitted β-particles is not discrete but is found to be continous.

➡ Therefore, in interpreting the β-spectrum, we must have to reconcile two facts

i) The existence of discrete energy level.

ii) The existance of continuous energy spectrum.

Difficulties in interpretation of β-spectrum :- The various difficulties in interpreting the β-Spectrum are —

i) The nuclear energy states are discrete. So the continues β-spectrum appears to be a violation of the conservation of energy principle. Attempts to explain the observed continuous distribution by way of loss of different amounts of energy due to scattering of β-particles in the source also failed.

ii) when a β-particle emitted with an energy W_K, the difference $W_m - W_K$ can not be accounted for.

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iii) The emitted β- particle also does not travel in the direction opposite to the recoil velocity of the daughter product. It thus appears to violate the principle of conservation of linear momentum also.

iv) The β- emission from a radio active nuclide is supposed to be the result of the transformation of either of the processes

$${}_1p^1 \rightarrow {}_0n^1 + {}_{+1}e^0$$

$${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$$

Since all the particles in the above two transformations are known to have half integral spin. The law of conservation of angular momentum also violated.

Neutrino hypothesis of Pauli :- Therefore a serial discrepency occur in interpreting the β- spectrum. The situation is tackled by Pauli by a daring postulate put forwarded by him. Pauli's idea was simply that in β-decay process, a second new particle was also simultaneously emitted.

To conserve the charge in the process, the new particle should be electrically neutral. Further β-particle is capable of carrying off all available energy. So, the new particle should carry little kinetic energy. Pauli postulated that the new particle had zero rest mass and zero charge.

To conserve the momentum, the new particle had to be endowed with a spin equal to $\frac{1}{2}\hbar$.

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This new particle was labelled neutrino and symbolised by ν and the Neutrino hypothesis of pauli.

● Explaination of β- spectrum using Neutrino hypothesis :-

The ~~nue~~ neutrino hypothesis explains well the emission of β- particles by transition of nucleon from neutron to the proton state with the simultaneous creation of an electron - neutrino pair. These two particles escape with a constant total energy, the maximum energy W_m available being equal to the difference between the energies of the original and the final nucleous.

The continuous energy distribution arises from the variable manner in which the total energy is shared between the electron and neutrino. The upper limit corresponds to the case where the neutrino gets no energy, the whole being carried off by the electron.

■ Neutrino :- Neutrino is a elementary particle. It is a fermion that interacts only via week interaction force. Thus neutrinos typically pass through normal matter un impeded and undetected. It is seen that two kind of neutrinos involved in β- decay. - Neutrino (ν) and Anti- ~~nue~~ neutrino ($\bar{\nu}$)

$$_{0}n^{1} \rightarrow {}_{1}p^{1} + {}_{-1}e^{0} + \bar{\nu}$$

$$_{1}p^{1} \rightarrow {}_{0}n^{1} + {}_{+1}e^{0} + \nu$$

● Properties of neutrino :-

i) It has no charge. Like neutron

ii) It has extreemely small mass

iii) It is a spin $\frac{1}{2}$ particle

iv) It interacts extreemely feebly with matter

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■ Neutrino and Anti-neutrino have the same properties - zero rest mass, zero charge and spin $\frac{1}{2}$. How can they be distinguished ??

⇒ Neutrino shows a handedness. While neutrino has its spin S_ν always anti parallel to its momentum P_ν, the spin $S_{\bar{\nu}}$ of an anti neutrino is always parallel to its momentum $P_{\bar{\nu}}$. Thus ν is left handed and $\bar{\nu}$ is right handed particle.

■ Energetics of β-decay :- In all the three processes of β-decay, namely β^- decay, β^+ decay and orbital electron capture the mass number A of the parent nucleus does not change only the atomic number changes by one unit.

β^--decay :- $${}_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0}$$

∴ The disintegration energy in β^- decay is

$$Q_{\beta^-} = [M_n(A,Z) - M_n(A, Z+1) - m_e]c^2$$

$$= [M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e]c^2$$

$$= [M(A,Z) - M(A,Z+1)]c^2$$

$$= M(A,Z) - M(A,Z+1) \text{ in Energy unit.}$$

∴ M_n → Nuclear mass

M_o → Atomic mass

m_e → mass of the e^-

$Q_{\beta^-} > 0$ when $M(A,Z) > M(A,Z+1)$

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This implies that β^- decay occurs only when the mass of the parent atom is greater than the mass of the daughter atom.

β^+ **decay** :-

$$_{Z}X^{A} \longrightarrow {}_{Z-1}Y^{A} + {}_{+1}e^{0}$$

$$\therefore Q_{\beta^+} = [M_n(A,Z) - M_n(A,Z-1) - m_e]c^2$$

$$= [M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e]c^2$$

$$= [M(A,Z) - M(A,Z-1) - 2m_e]c^2$$

$$= M(A,Z) - M(A,Z-1) - 2m_e \quad \text{in energy unit}$$

$$\therefore Q_{\beta^+} > 0 \quad \text{if} \quad M(A,Z) > M(A,Z-1) + 2m_e$$

This implies that β^+ decay is possible if the mass of the parent atom is greater than daughter atom by at least twice the electronic mass i.e $(2 \times 0.51\ MeV) = 1.02\ MeV$

Orbital electron capture :-

$$_{-1}e^{0} + {}_{Z}X^{A} \longrightarrow {}_{Z-1}Y^{A}$$

$$\therefore Q_e = [M_n(A,Z) + m_e - M_n(A,Z-1)]c^2 - B_e$$

$$= [M(A,Z) + m_e - Zm_e - M(A,Z-1) + (Z-1)m_e]c^2 - B_e$$

$$= [M(A,Z) - M(A,Z-1)]c^2 - B_e$$

$$Q_e = M(A,Z) - M(A,Z-1) - B_e \quad \text{in Energy unit}$$

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$\therefore Q_e > 0$ if $M(A,Z) > M(A, Z-1) + B_e$

This implies that electron capture is possible if the mass of the parent atom is greater than daughter atom by at least the binding energy of the electron.

Inverse β decay :- The β decay of a proton in a nucleus is given by

$$_1p^1 \rightarrow {_0n^1} + {_{+1}e^0} + \nu \quad \text{--- (i)}$$

The electron capture on the other hand is represented as

$$_1p^1 + {_{-1}e^0} \rightarrow {_0n^1} + \nu \quad \text{--- (ii)}$$

Essentially the equation (ii) is the same as (i) for the emission of a particle is equivalent to the absorption of its antiparticle.

Since the absorption of an antineutrino is equivalent to emission of a neutrino, the reaction

$$_1p^1 + \bar{\nu} \longrightarrow {_0n^1} + {_{+1}e^0} \quad \text{--- (iii)}$$

involves the same physical process as that of (i). The reaction (iii) is known as the inverse β-decay.

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Questions of β decay

1) By how much must the atomic mass of parent exceed the atomic mass of a daughter when (a) an electron is emitted (b) a positron is emitted and (c) an electron is captured.

2) The nuclide $_4Be^7$ is unstable and decays into $_3Li^7$ by electron capture. Why does it not decay by positron emission.

3) Show that it is energitically possible for Cu^{64} to undergo beta decay by e^- emission, positron emission and electron capture and find the energy release in each case.

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γ - ray

Origin of γ-rays :- It has been observed that γ-rays usually accompany α - and β emission. Origin of γ-ray can be easily explained on the basis of quantized energy states of the nucleus. Stable nuclides are usually in the ground state but they can be excited by particle or photon bombardment. One way an excited nucleus can get rid of its energy of excitation is by the emission of γ-rays. The γ-decay is represented schematically as

$$({}_{Z}X^{A})^{*} \rightarrow {}_{Z}X^{A} + \gamma$$

The γ-decay is governed by the principle of conservation of linear momentum and mass-energy. For instant if E^* be the energy associated with the excited state and E be the energy of the ground state then

$$E^{*} - E = h\nu$$

Where ν is the frequency of the emitted γ-photon.

If the decaying nucleus is initially at rest the ~~con~~ to conserve the linear momentum the daughter nuclei must recoil with a momentum P_N equal and opposite to the emitted γ photon.

$$\therefore P_N = \frac{h}{\lambda} = \frac{h\nu}{c}$$

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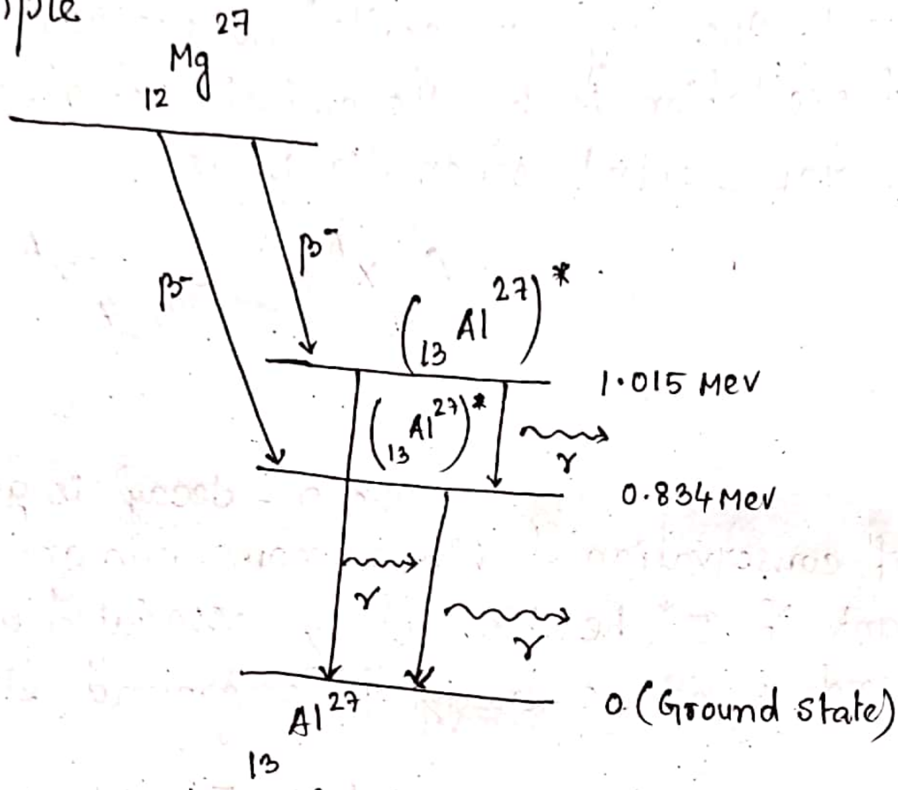
Let us take an example



Most excited nuclei have very short half life against gamma decay. But a few remain in excited state for as long as several hours. The long lived excited nucleus is called an isomer of the same nucleus in its ground state. The excited nucleus $(_{38}Sr^{87})^*$ has a half-life of 2.8 h and is accordingly an isomer of $_{38}Sr^{87}$.

<u>γ- ray spectrum</u> :- If the daughter nucleus finds itself in the excited state after the decay of the parent nucleus then the resulting γ rays emitted have well-defined energy corresponding to the two nuclear states involved in the transition. Such γ-rays are called characteristic γ-rays.

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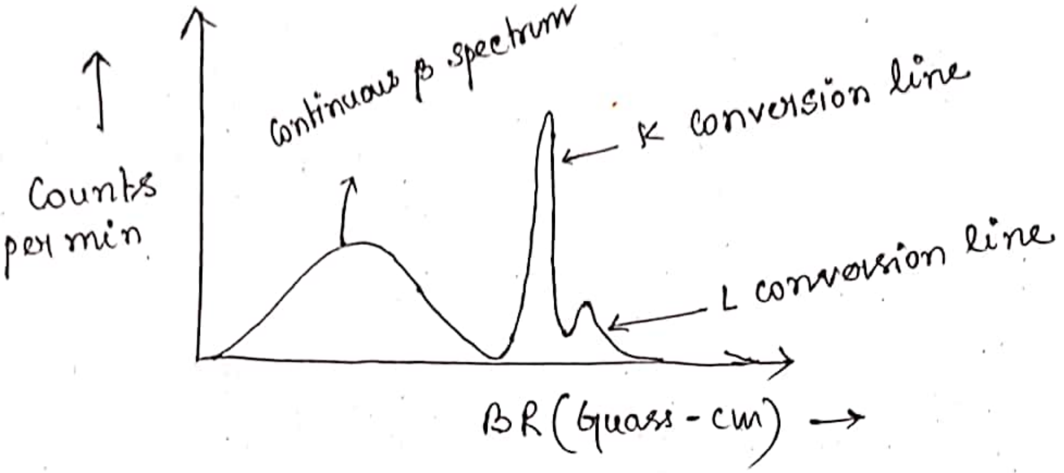
Apart from the sharp characteristic line spectrum of γ- rays there is also the continuous spectrum. Its origin is very similar to that of the continous x-ray spectrum. It is due to the radiation emitted when the β-particle gets accelarated while moving close to any nuclei.



- **Internal conversion :-** Sometimes it happens that the excited nucleus return to the ground state by giving up its excitation energy to the orbital electrons around it resulting electron emission. This phenomenon is known as internal conversion. If W be the excitation energy of the nuclei and W′ be the binding energy of the electron in the shell from which it is released, the kinetic energy of the emitted electron W_K is

$$W_K = W - W'$$

The differance in energy of the electrons released by a transition in the daughter nuclei should be equal to the difference between x-ray energy levels of the daughter nuclei.

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